

LIMIT LOADS FOR AN ANCHOR/TRAPDOOR EMBEDDED IN AN ASSOCIATIVE COULOMB SOIL

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SUMMARY

A methodology for determining the plane strain limit load acting on an anchor or trapdoor buried within a purely associative Coulomb soil is presented. True lower bounds derived from a family of limiting stress fields appropriate to shallow horizontal trapdoors and anchors are shown to correlate to within less than 1 percent of upper bounds available in the literature, permitting the true limit load to be almost exactly defined. The solution form alters for deeply buried anchors and trapdoors resulting in poorer correlations. Methods by which the work may be extended to cover the more practical instances of non-associative Coulomb soils are indicated but are beyond the scope of the current paper. © 1998 John Wiley & Sons, Ltd.

Key words: anchor; trapdoor; limit load; cohesionless; associative

1. INTRODUCTION

The anchor/trapdoor problem has many important applications in the design of such structures as buried pipes and culverts, ground anchors and silos. While the literature contains a large body of work relating to the analytical prediction of limit loads for such problems, these have for the most part involved several semi-empirical assumptions as to the nature of slip surfaces and stress distributions within the soil above the anchor/trapdoor. In deriving solutions, many previous authors,^{1–6} have assumed the shape of the failure surface based on experimental evidence, and from these, they made the stress computation. In contrast other authors^{7–10} performed an analysis of vertical pullout of anchors using the theory of cavity expansion or by using the finite element method.

Most authors distinguish between the different failure modes for shallow and deep anchors. Meyerhof¹¹ proposed a general solution of inclined shallow and deep anchors based on the earth pressure coefficients for an inclined wall.¹² Tagaya *et al.*⁷ proposed Meyerhof's solution for shallow anchors but introduced a new solution method for deep anchors based on the concept of cavity expansion. Several authors have approached the problem using the upper- and lower-bound theorems of plasticity. Davis¹³ presented upper-bound and complex lower-bound solutions for a trapdoor overlain by clay. Sloan *et al.*¹⁴ implemented lower- and upper-bound plastic analyses into a finite element numerical solution for a trapdoor overlain by clay. Murray and

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Geddes¹⁵ presented some simple upper- and lower-bound solutions to the anchor/trapdoor problem in sand.

The purpose of this paper is to present a novel rigorous limiting stress field solution to the anchor/trapdoor problem in a Coulomb soil (sand) and to demonstrate its validity by applying it to the associative case. Consideration of an associative material enables the problem to be considered on a purely theoretical basis by comparing lower- and upper-bound predictions. The application of these solutions to non-associative soils is also briefly considered. The solutions presented in this paper will be restricted to the plane strain case of a horizontal anchor/trapdoor.

1.1. Definition of the anchor/trapdoor problem

For the purposes of this paper, the anchor/trapdoor problem to be solved will be that as defined by Terzaghi¹⁶ and may be described as follows: with reference to Figure 1 trapdoor of width B is embedded within a rigid basement layer QQ' and overlain by a depth D of soil to the surface PPR' . The trapdoor is defined as *active* for downwards displacement, and *passive* for upwards displacement, the terms describing mechanisms in which the self-weight of the soil, respectively, assists or resists failure. The active case corresponds to such soil-structure interaction problems as the collapse of buried pipes and culverts. The passive case is related to the vertical anchor problem, though it differs from this situation in that a rising anchor plate leaves a void beneath itself. Where the soil is assumed to have uniformly distributed properties of self-weight (γ), angle of friction (ϕ) and angle of dilation (ν) then it is convenient to express the limit load (F) for the anchor or trapdoor in a dimensionless form relating load ratio ($F/\gamma DB$) to embedment ratio (D/B):

$$\frac{F}{\gamma DB} = 1.0 \pm \frac{D}{B} \Psi(\phi, \nu) \quad (1)$$

the sign depending on whether displacement is active (−) or passive (+).

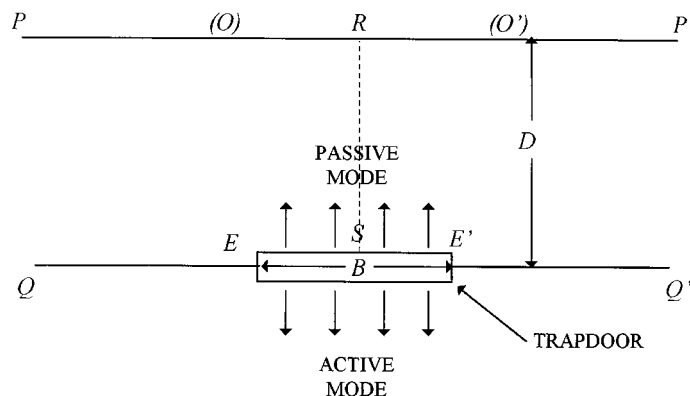


Figure 1. The trapdoor problem

For the remainder of this paper the term trapdoor will be used to indicate both an anchor (passive) and trapdoor (active). To avoid confusion in the use of the terms upper and lower bound (the upper bound kinematical method generates a physical lower bound limit load to the active trapdoor case and *vice versa*) the terms upper and lower bound will be used in the context of the passive (anchor) case.

2. ANALYSIS OF A SHALLOW TRAPDOOR

2.1. Upper-bound solution

Several authors, e.g. Murray and Geddes,¹⁵ have presented upper-bound sliding block solutions to the trapdoor problem considered here and the solutions are briefly presented for reference only. Figure 2 depicts the generic solution of interest. The upper bound is chosen by varying the geometry within bounds dictated by kinematic conditions until the maximum value of F is produced. A study of the passive case indicates that the optimum solution occurs when $\alpha = \phi$ and is independent of β while the optimum solution for the shallow active case, assuming $D/B < 1/(2 \tan \phi)$, occurs when $\beta = \phi$ and is independent of α . For these two cases, the function Ψ is given by

$$\Psi(\phi, v) = \tan \phi \quad (2)$$

where $v = \phi$ for the upper-bound theorem to apply.

2.2. Lower-bound solution

This section introduces a class of Sokolovskii¹² wedge-type limiting stress fields which can be applied to the trapdoor problem. The stress fields described are compatible with the presence of a displacing trapdoor underlying a body of cohesionless Coulomb-type soil and can be used to determine the failure load upon the trapdoor. Previous work on the trapdoor problem using the limiting stress field approach used incomplete stress fields, for example, adapted from retaining wall solutions^{17,11} which, while giving reasonable results, are not necessarily in equilibrium everywhere. In contrast, the stress fields presented here offer complete equilibrium solutions for both active and passive shallow trapdoors and assume limiting conditions everywhere.

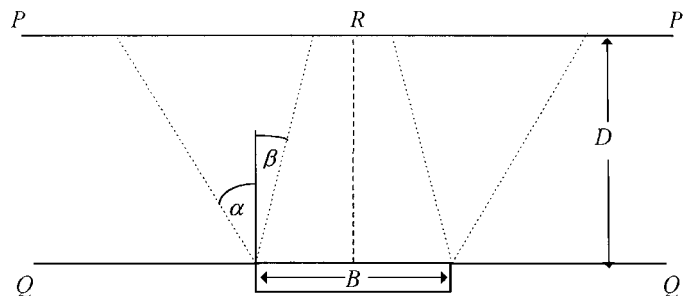


Figure 2. Upper-bound solution to the trapdoor problem

2.3. General considerations of the stress field above a trapdoor

Active case: Consider the downwards displacement of a trapdoor such as that depicted in Figure 1. In this case the soil immediately above the trapdoor will initially try to resist following the downward motion, leading to the vertical stress upon the trapdoor in the region of point *S* reducing from its at rest state to the limiting minimum value. Soil motion will then take place. As the soil follows the trapdoor movement, displacing inwards and downwards, it will tend to relieve the horizontal stresses in the soil either side of the trapdoor (arching). The major principal stress direction just below the surface near points *P*, *P'* may thus be considered to be vertical, while in the region of *R* the major principal stress direction should match that at *S* (horizontal) unless a stress discontinuity separates the two points.

Passive case: In a similar way, consideration of upwards displacement of the trapdoor leads to the reverse of the active case. Uplift of the soil by the trapdoor tends to push up and outwards on the adjacent soil mass, leading to the major principal stress acting vertically in the region between *RS* and horizontally in the region of *P* and *P'*.

The attributes of a limiting stress field *compatible* with an underlying displacing trapdoor within a rigid basement layer may thus be stated as follows:

- (1) The orientation of the principal stresses within the soil just beneath the soil surface can only be vertical or horizontal and must differ for soil lying directly above the trapdoor (e.g. in the region of *R* in Figure 1) and for soil lying some distance to the side (e.g. in the region of *P*). A transition zone (in the region of *O*) must therefore exist between the two.
- (2) The direction of shear stresses along, for example, a line joining *O* and *E* in Figure 1 must act as to oppose the soil motion.

In order to proceed with the analysis it is necessary to restrict it to the case where the soil surface is free of loading; i.e. stresses within the soil are due to the soil self-weight only.

For the purposes of the lower-bound analysis, a shallow trapdoor will be defined as one buried at a sufficiently small embedment ratio such that the limiting soil stress fields generated at each edge of the trapdoor do not interact. A complete analysis will thus require the investigation of the stress field at one edge only, as shown in Figure 3, and will be applicable to both active and passive trapdoors, depending on whether point *E* is taken as the left- or right-hand edge of the trapdoor. Under these conditions the problem is reduced from one that has two characteristic lengths (depth and trapdoor width) to one that has only one characteristic length (depth) and since there is no surface loading, there is also no characteristic stress. The only possible limiting stress field for this dimensionless problem compatible with the required attributes above is therefore one that is Rankine-type outside the transition zone and inside is radially linear with origin (*O*) on the surface. Such a field has been discussed by Sokolovskii¹² in the analysis of the equilibrium of a cohesionless wedge. Following Sokolovskii, the equations characterizing a radially linear stress field may be derived as follows.

The equations of equilibrium in polar co-ordinates can be written as

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = \gamma \sin \theta \quad (3)$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\tau_{r\theta}}{r} = \gamma \cos \theta \quad (4)$$

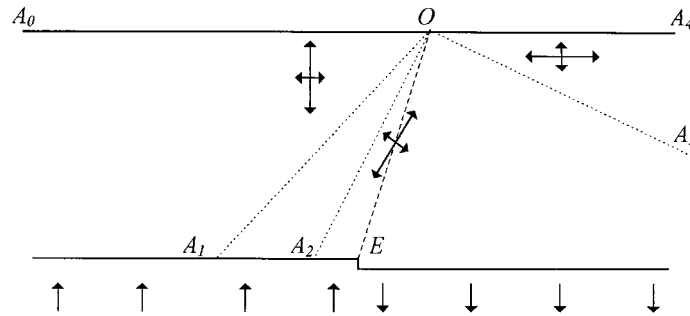


Figure 3. Principal stress orientations and discontinuities above a single discontinuity in the basement layer

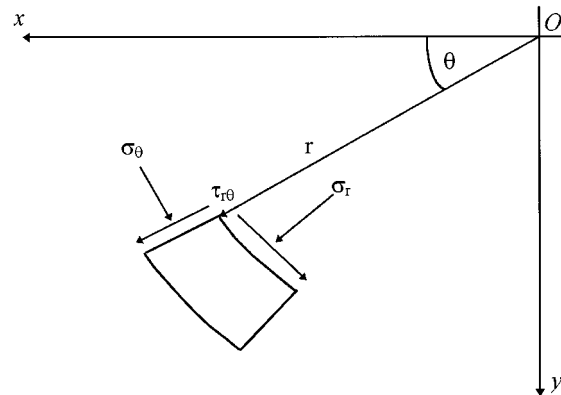


Figure 4. Stress components in polar co-ordinates

where normal and shear stresses are taken as negative in the positive r and θ directions (compressive stresses positive) as indicated in Figure 4. The Coulomb yield conditions may be written as

$$\left. \begin{matrix} \sigma_r \\ \sigma_\theta \end{matrix} \right\} = \sigma(1 \pm \sin \phi \cos 2\psi) \quad (5)$$

$$\tau_{r\theta} = \sigma \sin \phi \sin 2\psi \quad (6)$$

where ψ is the angle between the principal stress direction and the radial vector (positive anticlockwise), ϕ is the soil friction angle, and σ is the mean stress.

Defining

$$\sigma = \gamma r \chi(\theta) \quad (7)$$

where $\psi = \psi(\theta)$ and $\chi = \chi(\theta)$ permits the equations of equilibrium and yield to be solved, giving the following two ordinary differential equations:

$$\frac{d\chi}{d\theta} = \frac{\cos(2\psi + \theta) + \chi \sin 2\psi}{\cos 2\psi - \sin \phi} \quad (8)$$

$$\frac{d\psi}{d\theta} + 1 = \frac{\sin \theta - \sin \phi \sin(2\psi + \theta) - \chi \cos^2 \phi}{2\chi \sin \phi (\cos 2\psi - \sin \phi)} \quad (9)$$

which can be solved straightforwardly by numerical methods with appropriate boundary conditions. The stress characteristics consist of two families intersecting each other at an angle $2\varepsilon = (\pi/2) - \phi$, and are inclined to the radial vector at an angle $(\psi \mp \varepsilon)$. The differential equations of the characteristics can thus be written as

$$\frac{dr}{d\theta} = r \cot(\psi \mp \varepsilon) \quad (10)$$

To obtain a limiting stress field with the required attributes of the trapdoor problem, it has been found that it is necessary to introduce a minimum of three stress discontinuities along the lines OA_1 , OA_2 and OA_3 as shown in Figure 3 with the conditions that

$$0 < \theta_1 < \frac{\pi}{2} - \varepsilon \quad (11)$$

$$\theta_1 < \theta_2 < \theta_3 \quad (12)$$

$$\pi > \theta_3 > \pi - \varepsilon \quad (13)$$

where the angles θ_1 , θ_2 , θ_E and θ_3 are measured anticlockwise from the line OA_0 to the lines OA_1 , OA_2 , OE and OA_3 , respectively.

A limiting stress field of this nature has one degree of freedom; for every distinct value of θ_2 that can be chosen, there exists a correspondingly distinct equilibrium limiting stress field. Given a specified value of θ_2 , values of θ_1 and θ_3 may be at first guessed. Since the two outer limiting regions OA_0A_1 and OA_4A_3 contain Rankine stress states, it is possible to determine χ and ψ on OA_1 and OA_3 . With these boundary conditions, equations (8) and (9) can be solved numerically from both OA_1 and OA_3 to the line OA_2 using a suitable integration scheme. It is then necessary to iterate for both θ_1 and θ_3 until equilibrium values of χ and ψ are produced on OA_2 .

An example solution to the shallow passive trapdoor problem for $\theta_2 = 61.7^\circ$, $\phi = 30.0^\circ$ can be seen in Figure 5 in the vicinity of O. Both principal stresses and stress characteristics are presented. The stress characteristics can be seen to be geometrically similar and to scale linearly with distance from O and the principal stress directions can be seen to rotate as required. The above analysis is unusual in that it is not possible to relate immediately the position of a trapdoor edge in a rigid basement layer to the given stress field which was solved from the surface downwards alone and thus has made no direct consideration of the rigid basement containing a trapdoor. The stress field should thus be regarded as a solution upon which a rigid basement can be superimposed at an arbitrary depth so delineating the stress field's lower boundary. Strictly, it is therefore necessary to require that this rigid base is rough with a maximum interface angle of friction ϕ in order that the derived field is everywhere in equilibrium. The position of E will be defined by the intersection of the radius OE with the superimposed basement layer. Thus, the stress field solution for a trapdoor has two degrees of freedom, one in θ_2 defining the stress field and one in θ_E defining the trapdoor position. It is postulated that each trapdoor stress field defined by the pair of values θ_2 and θ_E may correspond to the case of a trapdoor at a particular inclination underlying a soil of a particular dilatance and surface slope. If it is

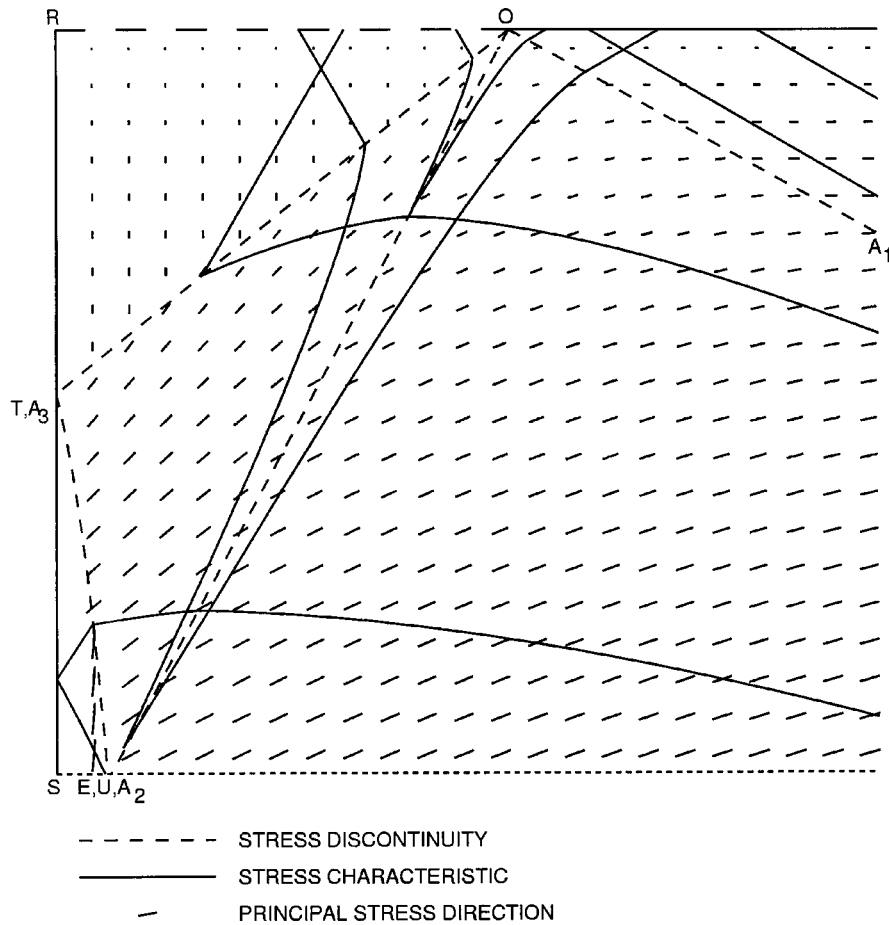


Figure 5. Principal stress directions (relative magnitudes exaggerated) and stress characteristics for a limiting stress field applicable to a shallow passive trapdoor ($\phi = 30^\circ$)

assumed that the position of OE or the value of θ_E is known, then the force upon the trapdoor may be computed. It is most straightforward to compute this by integrating the normal and shear stresses along the boundary OE to give the following expression for the overall upward vertical force exerted along the boundary:

$$F_u = \frac{\gamma \chi R^2}{2} \sin \phi \cos(2\psi + \theta_E) - \cos \theta_E \quad (14)$$

Determination of the total normalized downward force $F/\gamma DB$ on a trapdoor EE' , involves combining the action of the vertical forces F_u with the self-weight of the trapezoidal volume of soil $OEE'O'$ to give the following equation for Ψ :

$$\Psi(\phi, \theta_E, \theta_2) = \cot \theta_E + \frac{\chi}{\sin^2 \theta_E} (\sin \phi \cos(2\psi + \theta_E) - \cos \theta_E) \quad (15)$$

Table I. Lower- and upper-bound solutions to the shallow trapdoor problem (all angles given in degrees) for given angles of friction. (Discontinuity angles θ_1 , θ_2 , and θ_3 are defined in Section 2 and Figure 3. Limit load function Ψ is defined in equation (1))

ϕ	Discontinuity angles			Ψ_{lower}	$\Psi_{\text{upper}} = \tan \phi$	D/B limit of validity	
	θ_1	$90-\theta_2$	$180-\theta_3$			Passive	Active
25	39.39	23.67	30.95	0.4650	0.4663	5.6	0.87
30	38.59	28.32	28.86	0.5754	0.5774	7.2	0.74
35	37.45	32.95	26.65	0.6974	0.7002	9.6	0.63
40	35.98	37.57	24.37	0.8349	0.8391	13	0.55
45	34.15	42.21	22.03	0.9940	1	17	0.47
50	31.94	46.90	19.65	1.1831	1.1918	28	0.40

where χ and ψ are functions of θ_2 and ϕ . With reference to equation (1), θ_E , θ_2 may be taken as functions of v . The above analysis is applicable to both associative and non-associative soils. In the case of an associative Coulomb soil for which the dilation angle v is equal to the friction angle ϕ , the lower-bound theorem of plasticity may be applied and the optimum solution for both active and passive trapdoors may be found in the case where $\Psi(\phi, \theta_E, \theta_2)$ is a maximum. The variation of $\max(\Psi(\phi, \theta_E, \theta_2))$ with θ_E is found to agree within 1 per cent of $\tan \phi$ (the upper-bound solution, equation (2)), and θ_E is found to exactly equal θ_2 and to agree within a few degrees of $90^\circ - \phi$ (typical results are given in Table I). However, they are *not* identical. This would indicate that while upper and lower bound almost exactly correspond, there is a better solution to one or the other. The solution is valid up to the point at which the stress fields at each edge interact at the centre of the trapdoor (point T in Figure 5) at which point the problem may be classed as one of a deep trapdoor. Since the stress fields used in this solution method are all radial in nature, then they may be extended laterally to infinity away from the trapdoor. Hence, the conditions for the application of the lower-bound theory are satisfied. Equilibrium, yield and the stress boundary conditions have been satisfied throughout the domain as defined in Section 1, with the additional constraint (discussed above) where it was required that the rigid basement layer is rough with a maximum interface angle of friction ϕ .

3. ANALYSIS OF A DEEP TRAPDOOR

3.1. Upper bound

The solution for the passive case is identical to that for a shallow trapdoor. For a dilative soil a kinematically admissible solution for the active case for $D/B > 1/(2 \tan \phi)$ is one involving a single displacing isosceles triangle of soil whose base is defined by the trapdoor and whose apex angle equals 2ϕ . No deformation reaches the surface. Results for the case where $\phi = 30^\circ$ are presented in Figure 6. It is seen that while the limit loads for the active case are defined by a hyperbola, the passive increases linearly to infinity. The latter is not seen in reality.

3.2. Lower bound

The shallow trapdoor problem was defined by assuming that the limiting stresses at either side of the trapdoor did not interact. In circumstances where this interaction begins to be significant,

the stress fields must be modified (solving by the method of characteristics) so that the shear stresses along the central line of symmetry remain zero, giving rise to what can be termed a 'deep' solution. Initially, this occurs when discontinuities OA_1 in the active case (or OA_3 in the passive case) meet on the line of symmetry of the trapdoor (point T in Figure 5). In order to maintain equilibrium these lines steepen in gradient and the solution ceases to be definable purely in terms of a radial stress field. Up until the point at which these lines meet discontinuities OA_2 (point U in Figure 5), the shallow trapdoor solution remains valid as a lower bound i.e. the locus of point E may be defined as the line OU . The largest valid embedment ratio for the shallow case is defined by the position of U and is given in Table I. Thereafter better lower-bound solutions may be found based on radial stress fields with larger values of θ_2 for the active case and smaller values of θ_2 for the passive case. In each of these cases the optimum solution occurs when point E coincides with point U . Since there is a limit to the range of θ_2 values that produce a radial solution, there is also a limit to the range of embedment ratios for which this deep solution can be valid. It was not possible to ascertain these precisely due to limits in computational precision. Lower bounds for both passive and active cases are presented in Figure 6 for $\phi = 30^\circ$ with the upper-bound solutions for comparison.

An alternative, simpler solution form for deep trapdoors can be derived, again based on a single radial stress field, by assuming that the origin O of the field lies on the line of symmetry. An example of the stress characteristics for an active 'deep' trapdoor with an embedment ratio of 6 is illustrated in Figure 7. In this case, the stress characteristics can again be seen to be geometrically similar and to scale linearly with distance from O . The optimum solution occurs when E lies on the line OA_2 as before. This solution form does not generate better solutions than the above more complex method, however, it is possible to generate solutions for all embedment ratios from approximately 1 upwards in the active case and 2 upwards in the passive. This may be used to continue the lower bounds when the complex solution reaches the bounds of its limits.

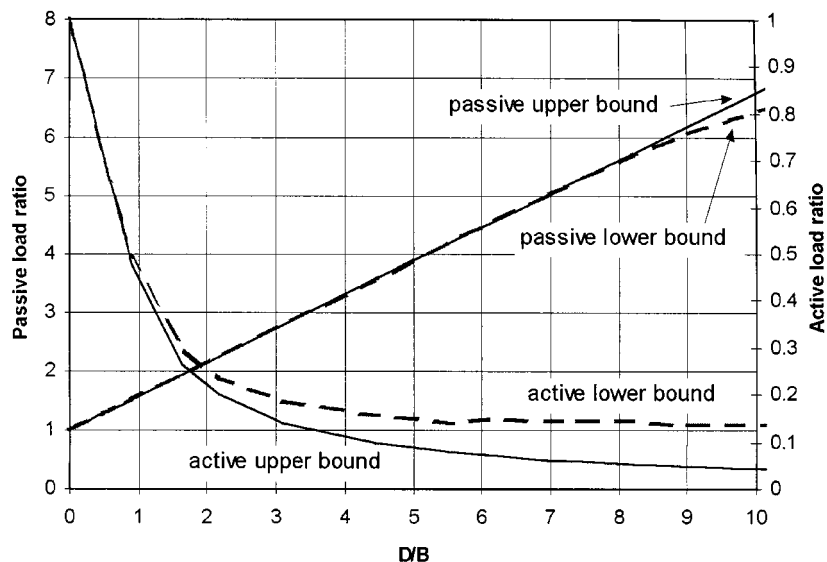


Figure 6. Upper and lower bound solutions to the trapdoor problem for an associative coulomb soil ($\phi = 30^\circ$)

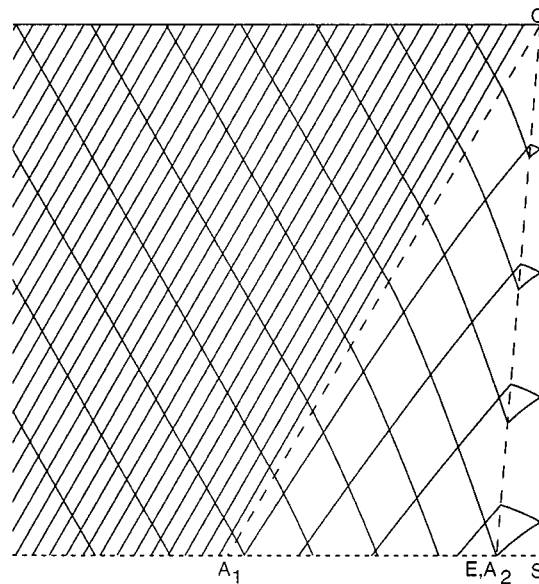


Figure 7. Stress characteristics for a limiting stress field applicable to a deep active trapdoor (embedment ratio = 6, $\phi = 30^\circ$)

4. DISCUSSION

The aim of this paper was to establish the above limiting stress field solutions as being of significant relevance in the analysis of the trapdoor problem by considering the limit loads for the associative case in a theoretically rigorous manner. It is reassuring to note that in all solutions, the edge of the trapdoor E coincides with the central stress discontinuity as might be expected even though this arose purely from selecting the case which gave the maximum or minimum loading as required. It is also reassuring to note that, if, for the associative material, the stress characteristics are regarded as representative of the velocity characteristics, then the patterns appear logical and match qualitatively with what is seen in experiment. This gives confidence that the solutions are of significant applicability to the trapdoor problem.

While the usefulness of the present results is probably restricted to the calibration of numerical models, the work may be extended to cover the more practical instances of non-associative coulomb soils. The limiting stress field (lower bound) solution may be adapted to a non-associative case by using a pseudo-kinematic condition at the edges of the trapdoor derived from observations by Vermeer and Sutjiadi,¹⁸ while the kinematic (upper bound) solution can be adapted using a limit load theorem developed for translation mechanisms in non-associative materials.¹⁹ Presentation of these modifications, together with a comparison with experimental results is reserved for a follow up paper. Some consideration of the adaptation of the limiting stress field solution is also given by Smith²⁰ who additionally gives a more detailed presentation of results for shallow and deep modes and discusses the effects of trapdoor inclination and surface slope. In this work the adapted method is shown to give good correlations with experimental results reported in the literature.

5. CONCLUSIONS

- (1) A class of complete equilibrium limiting stress fields that can be used in the solution of the trapdoor problem in a cohesionless Coulomb soil has been presented. The analysis makes use of stress discontinuities and radial stress fields, and can be used to predict the forces on both active and passive trapdoors.
- (2) Three distinct though related solution forms are presented which allows solutions to be defined at all embedment ratios.
- (3) In the case of associative soils, the lower-bound solutions for the shallow case match upper-bound solutions in the literature to within better than 1 per cent, thus effectively defining the true limit load.
- (4) Optimum lower-bound solutions generate intuitively correct solutions with regard to stress characteristics and stress discontinuities giving confidence in the method.
- (5) The potential application of the method to non-associative soils has been discussed.

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